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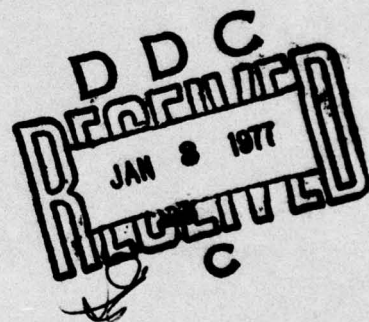UNCLASSIFIED   NRL-MR-3406   NL

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The Initial Behavior of Plasmons in the Presence of a Long Wavelength Large Amplitude Phonon

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Plasma Physics Division

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Using a one dimensional two-fluid model the behavior of electrostatic electron plasma oscillations in the presence of a large amplitude long wavelength ion acoustic wave has been studied. The electron waves are observed to collect initially in regions of maximum density gradients. The initial increase in the variance of the plasmon wavenumbers is quadratic in time. When the high frequency fields are characterized by a narrow spectrum in k-space, a solution of the fluid equations is obtained which exhibits these results.

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THE INITIAL BEHAVIOR OF PLASMONS IN THE PRESENCE OF A LONG WAVELENGTH LARGE AMPLITUDE PHONON

During the last few years there has been extensive examination of the nonlinear system of Langmuir oscillations. Motivation for these studies is their relevance to plasma heating via powerful relativistic electron beams or powerful coherent laser radiation. A particularly striking effect is the observation in computer simulations of the localization of electrostatic fields accompanied by self-consistent density cavities. Generally, the theories of this phenomenon have involved the nonlinear Schrodinger equation and its envelope soliton solutions.

To complement these studies we have made numerical studies of the two-fluid plasma equations. We initialize the electron fluid with a set of long wavelength electrostatic waves ($k_e < k_D/3$, where k_e is the initial plasmon wavenumber and k_D is the Debye wavenumber). Inhomogeneities in this field will lead to the development of envelope Langmuir solitons. The following heuristic description of the process is from a note by Nishikawa et al.[1]

> Suppose a small amplitude modulation is introduced to the uniform plasma oscillations. Then some of the plasma density is expelled from the larger amplitude region by the pondermotive force. As a result, the local plasma density, and hence the local plasma frequency is decreased in the larger amplitude region. A decrease of plasma frequency implies an increase of the "wavenumber", which means a further localization of the wave field. In this way, the small-amplitude modulation is amplified.

The objective of our calculations is to numerically study this process. To get our system going, we initialize the ion fluid with

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a long wavelength sinusoidal ion acoustic wave ($k_s \approx k_e/20$, where k_s is the wavenumber of the acoustic wave). Our computations show a localization of the Langmuir radiation. However, in contrast to the above description, the observed decrease in the wavenumber is initially localized in regions of strong plasma density gradients. We also measure the variance in the Fourier wavenumbers during the localization process. This variance initially proceeds quadratically in time, not linearly which would be the case if the wave amplitudes were following a diffusion equation in k-space.

These results are understood by noting that in our computations we are solving an initial value problem and that for our initial condition the high frequency fields have a negligible group velocity. For this condition we find that the fluid equations may be solved analytically, and that the solution exhibits the above characteristics.

As the Fourier wavenumbers spread, the amount of frequency variation in the spectrum of the high frequency waves increases. In our simulations we then observe the localization to shift to regions of low plasma density and the development of envelope wave structures. Initially the effect of the density modulation is a dynamic process of wave localization. Later the system evolves so that the density modulation is balanced by the dispersion or pressure associated with a wave amplitude modulation.

We will first describe our numerical model. We will then relate our results and discuss them in terms of the fluid equations.

Our model consists of the one-dimensional fluid equations for the electrons and ions,

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$$\frac{\partial n_e}{\partial t} + \frac{\partial}{\partial x}(n_e v_e) = 0, \tag{1}$$

$$\frac{\partial v_e}{\partial t} + \frac{\partial}{\partial x}(v_e^2/2 - \phi + \gamma_e \ln n_e) = 0, \tag{2}$$

$$\frac{\partial n_i}{\partial t} + \sqrt{\frac{m_e}{m_i}} \; \frac{\partial}{\partial x}(n_i v_i) = 0, \tag{3}$$

$$\frac{\partial v_i}{\partial t} + \sqrt{\frac{m_e}{m_i}} \; \frac{\partial}{\partial x}(v_i^2/2 + \phi + \gamma_i \frac{T_i}{T_e} \ln n_i) = 0 \tag{4}$$

and Poisson's equation,

$$\frac{\partial^2 \phi}{\partial x^2} = n_e - n_i. \tag{5}$$

The independent variables x and t have been scaled to the inverse electron Debye wavenumber k_D^{-1} and the inverse electron plasma frequency ω_p^{-1} respectively. The subscripts e and i refer to electron and ion properties respectively; m is mass, T is temperature, and γ is the ratio of specific heats. The densities, n_e and n_i, are scaled to the equilibrium density n_o; the velocities, v_e and v_i, are scaled to the electron thermal velocity ω_p/k_D and the ion acoustic velocity $(\omega_p/k_D)\sqrt{m_e/m_i}$ respectively; and the potential ϕ is scaled to the electron temperature in volts. For the calculations reported here we have chosen an isothermal equation of state for the electrons

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$\gamma_e = 1$, and assumed that the ions are cold $T_i = 0$.

The fluid equations are in Eulerian form and are solved explicitly. The densities and velocities are advanced in time using a modified Lax-Wendroft two-step procedure.[2] In the spirit of Roberts and Weiss,[3] we use a fourth order algorithm for the derivatives of the fluxes and a small Courant parameter, $\Delta t/\Delta x = 0.1$. The algorithm is second order in time and fourth order in space. Poisson's equation is solved utilizing the FFT.[4]

Pereira[5] has also solved these equations numerically in his analysis of Langmuir solitons. His objective was also to compliment work involving the nonlinear Schrodinger equation. He observed the propagation of these solitons, their collisions, and their etiology in the oscillating-two-stream instability.

We initialize the ions as a travelling ion acoustic sinusoidal potential wave with a flow velocity equal to the ion acoustic velocity in the direction opposite to that of the wave. Disregarding non-linearities, the result is a time invariant ion density modulation,

$$n_i = 1 + \delta n_i \sin(k_s x). \tag{6}$$

The ion wave has a wavelength of 400 Debye lengths (i.e. $k_s/k_D = 2\pi/400$) and therefore there will be a low frequency electron density fluctuation approximately equal to this. In addition, the electrons are given a shorter wavelength high frequency fluctuation. For example, Fig. 1(a) shows the initial electric field which results when the ion wave has

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an amplitude of twenty percent, $\delta n_1 = 0.2$, and Fourier mode 19 is excited as a right traveling electrostatic electron plasma wave with a 0.1% amplitude, $\delta n_e = 0.001$. (The electric field is defined as $\partial \phi / \partial x$ and is therefore scaled to $\sqrt{4\pi n_o T_e}$.) For reference the discrete Fourier transform of this wave is also shown. The only nonzero components are the first, the phonon, and the nineteenth, the plasmon.

Figure 1(b) shows configuration space and k-space at time $t\omega_p = 108$. The plasmon has evolved to have shorter wavelengths in the center of configuration space. In k-space the phonon is unchanged, but the plasmon is now represented by a broad spectrum of modes. If the wave is launched to travel in the opposite direction, the mode spectrum at this time is the same, but the localization is centered around the origin in configuration space.

We were interested in whether or not we were observing a diffusion process in k-space so we recorded the spread of the Fourier wave-numbers of the plasmon. This is defined

$$\Delta k = \sqrt{< k^2 - < k >>}$$ (7)

where

$$< k^n > = \sum_k (k^n |E_k|^2) / \sum_k |E_k|^2 ,$$ (8)

with the sum extending over the plasmon wavenumbers.

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The time history of this spread for the case we have been con-
sidering is shown in Fig. 2. For a scattering process which can be
represented by a diffusion equation in k-space, the square of the
spread $(\Delta k)^2$, the variance, will be proportional to time. In contrast
Fig. 2 shows Δk to be approximately proportional to time. Furthermore,
the diffusion theory of Davidson and Goldman[6] indicates that the
plasmon-phonon scattering should proceed as $(\delta n_i)^2$. The process we
are observing, however, evolves linearly in δn_i. For example, Fig. 1(b)
is also the result for $\delta n_i = 0.1$ (half the phonon amplitude) at time
$t\omega_p = 216$ (twice the time) with the other parameters unchanged.

The results of this computation show that the high frequency
Fourier modes have approximately the same frequency and thus maintain
a constant phase relation during the localization process. This is
neglecting times when the amplitudes change sign, then there are
phase jumps of magnitude equal to π. With this in mind it is a
straightforward matter to obtain these results analytically.

The electron fluid equations, Eqs. (1) and (2), may be linearized
and combined to yield the following equation,

$$\frac{\delta^2 n_e}{\delta t^2} + (1 + \delta n_i \sin k_s x)\left(n_e - \gamma_e \frac{\delta^2 n_e}{\delta x^2}\right) = 0 \qquad (9)$$

We assume that the initial plasmon distribution is sharply peaked in
k-space so that the last term may be written $\delta^2 n_e/\delta x^2 \equiv -k_e^2 n_e/k_D^2$.
The resulting equation may be solved directly. Using Poisson's

equation the high frequency electric field is given by

$$E(x,t) = E_o(x) \exp \left[i(1 + \frac{\delta n_i}{2} \sin k_s x)\omega_{po}t \right] \qquad (10)$$

where ω_{po} is the frequency of the initial electric field $E_o(x)$,
$\omega^2_{po} = \omega^2_p (1 + \gamma_e k_e^2/k_D^2)$. (We have assumed that δn_i is small so
that $\sqrt{1 + \delta n_i} \simeq 1 + \delta n_i/2$.) If the initial condition is a right
traveling plasmon, the solution is

$$E(x,t) = E_o \sin \left[k_e x - (1 + \frac{\delta n_i}{2} \sin k_s x)\omega_{po}t \right] \qquad (11)$$

We can define a spacially dependent effective wavenumber k_{eff}
as the partial of the argument of the sine with respect to displacement.
This gives

$$k_{eff} = k_e - k_s \frac{\delta n_i}{2} (\cos k_s x)\, \omega_{po}t. \qquad (12)$$

This wavenumber increases (the plasmon localizes) near the center
of our configuration space where $k_s x = \pi$. This is the situation of
Fig. 1(b). Similar analysis indicates that a left traveling plasmon
will localize near the origin as we observe.

Returning to Eqs. (7), (8) and (10), using the Bessel generating
function, and an identity in Watson,[7] it is straightforward to show
that

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$$(\Delta k)^2 = [\Delta k(t = 0)]^2 + \frac{1}{2}\left(\frac{\delta n_i k_s}{2} \omega_{po} t\right)^2 \tag{13}$$

For the initially monochromatic case this result may be obtained directly from Eq. (12) by averaging over space. In Fig. 2 this line has been added as the dashed line. The spread in the Fourier wave-numbers initially increases according to this equation.

A better understanding of the assumption made in this analysis is obtained by substituting our solution for the high frequency density fluctuation into Eq. (9). We find terms proportional to powers of time which do not cancel since the electron wave does not remain monochromatic. The lowest order term in the discrepancy has a magnitude equal to $\gamma_e k_e (\delta n_i k_s \omega_{po} t)/k_D^2$. We have retained the effect of the density modulation and neglected this term which is proportional to the spread in k-space. We have made the following assumption

$$\gamma_e k_e (\Delta k)/k_D^2 < \delta n_i/2 \ . \tag{14}$$

The left hand-side of this inequality may be written as $\Delta k \ \partial \omega(k = k_e)/\partial k$ which is a frequency spread due to a wavenumber spread. When this spread is small relative to that caused by the density modulation then our analytic solution may be applied.

We wish to point out that Eq. (12) predicts another interesting phenomenon. At the time $t\omega_{po} = 2k_e/(k_s \delta n_i)$, the effective wavenumber

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k_{eff} will go to zero at the origin. Henceforth, the plasmon will be trapped; i.e., it will be reflected so as to be confined to certain regions of configuration space. In our simulations we have observed this but our understanding here is limited. Therefore, another limit to this theory is that $\Delta k < k_e$.

Our essential assumption is written as the inequality of Eq. (14). This led to a solution, Eq. (10), which in turn indicated that the spread of the Fourier wavenumbers is monotonically increasing in time, Eq. (13). Our theory breaks down after a period of time, and is therefore valid only in indicating the initial response of the system.

Figures 1(c) and 1(d) show the electric field and its Fourier spectrum for a smaller ion wave, $\delta n_i = 0.05$, somewhat later in time. In Fig. 1(c) the high frequency field was again initialized as a right traveling plasmon with an amplitude $\delta n_e = 0.001$. The wave collapse was again initially near the center of our configuration space, but at $t\omega_p \approx 250$ it began slowly moving to the right. This is evidence of a balancing of the last two terms of Eq. (9). The electron fluid of Fig. 1(d) was initialized with modes 16-22 excited as standing waves, all with an amplitude of $\delta n_e = 0.001$. Again we observe a balancing of the final terms of Eq. (9). In this case, with several Fourier components traveling in either direction, there is clearly an envelope structure to the waves in configuration space. Its shape and position correlate with that of the ions ($|E|^2 \propto - \delta n_i$).

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In this note we have reported the results of our numerical analysis of the one-dimension two-fluid plasma equations. Our computations are formulated as an initial-value problem and the initial behavior of our system of Langmuir oscillations seems to contrast with well-accepted notions. In particular, we observe the Langmuir waves to collapse or localize into regions of strong density gradients and not into density depressions. This is understood by noting that our particular choice of initial conditions has a narrow spectrum in k-space. As such, variation in the electron pressure term may be neglected in solving for the initial behavior. If this is done the fluid equations have a simple analytic solution which agrees with the results of our simulations. The solution indicates that the width of the plasmon spectrum in k-space will increase in time. Later in time, after our analytic solution becomes invalid, the position of the localization moves into the density depression. The system evolves into a state in which the density gradients are balanced by the electrostatic force and the force of spatially varying high frequency electric field. These later states exhibit behavior which agree with the well-established theoretical notions.

Acknowledgment

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References

1. K. Nishikawa, Y. C. Lee, and C. S. Liu, Comments Plas. Phys. $\underline{2}$, 63 (1975).

2. R. D. Richtmyer and K. W. Morton, <u>Difference Methods for Initial-Value Problems</u>, p. 303, Wiley Interscience, New York, 1967.

3. K. V. Roberts and N. O. Weiss, Math Comput. $\underline{20}$, 272 (1966).

4. R. C. Singleton, Comm. Assoc. Comp. Mach. $\underline{10}$, 647 (1967).

5. N. R. Pereira, Ph.D. Thesis, Cornell U., (1976); N. R. Pereira, R. N. Sudan, and J. Denavit, to be published in Phys. Fluids.

6. R. C. Davidson and R. Goldman, Phys. Rev. Lett. 21, 1671 (1968).

7. G. N. Watson, <u>A Treatise on the Theory of Bessel Functions,</u> 2nd ed., p. 37, University Press, Cambridge, England, 1962.
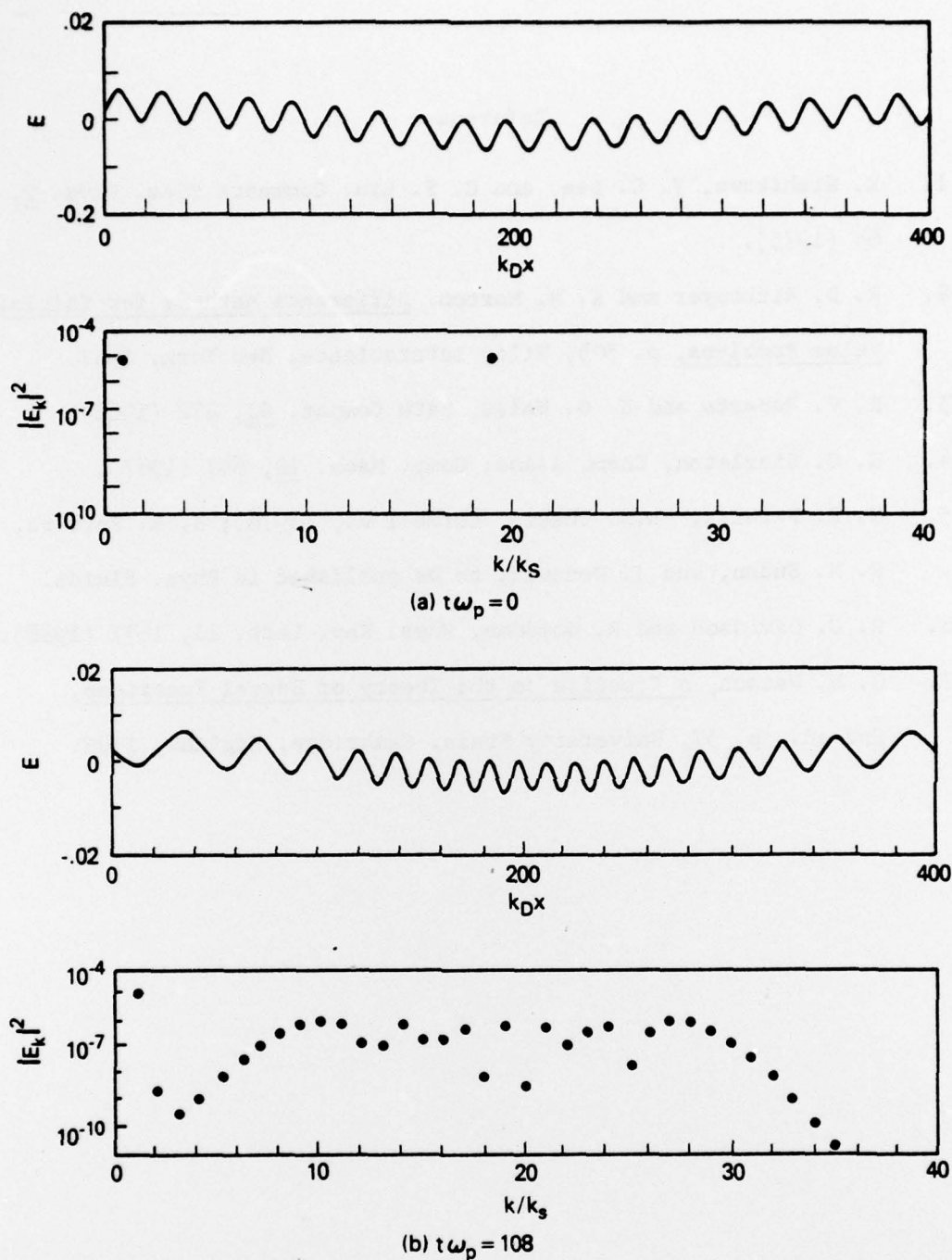
Fig. 1 — The electric field and its Fourier transform. (a) An initial condition: $n_i = 0.2 \sin k_s x$, $k_s = 2\pi k_D/400$, $n_e = 0.001 \sin (k_e x - \omega_p t)$, $k_e = 19 k_s$. (b) This is later in the same calculation, $t\omega_p = 108$. The collapse of the Langmuir radiation is evident.
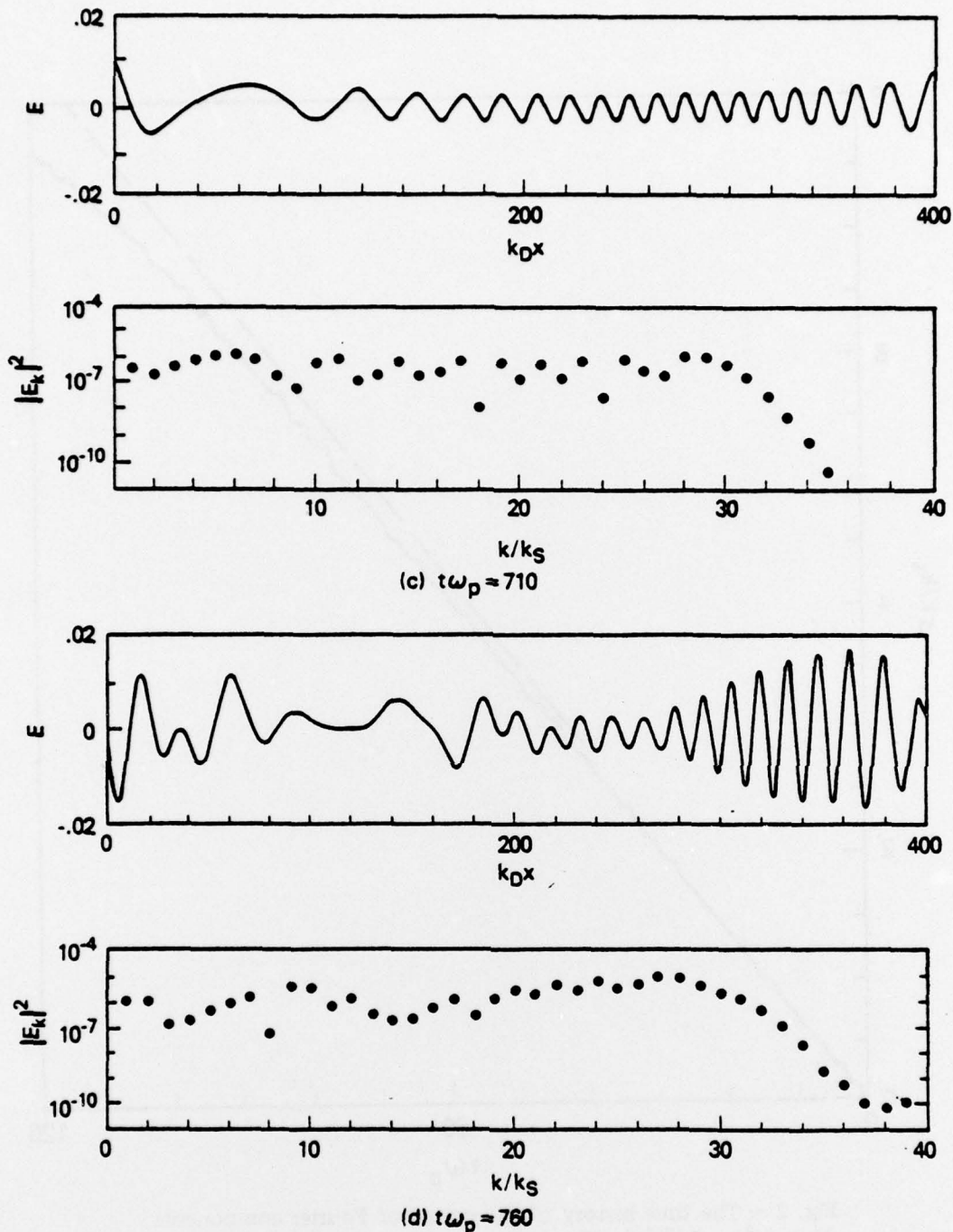
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Fig. 1 (Continued) — The electric field and its Fourier transform. (c) In this calculation the ion wave amplitude is reduced to $n_i = 0.05$; the electron fluid has the same initial condition. This is at $t\omega_p = 710$; the collapse has shifted to the right. (d) Again the ion wave has an amplitude of 5%, but now the electrons were initialized with a spectrum of standing waves, $n_e = 0.001 \sum_{\ell=16}^{22} \sin(\ell k_s x - \theta_\ell)$. ($\theta_\ell$ are randomly chosen phase angles.) This is at $t\omega_p = 760$; the field clearly exhibits an envelope structure.
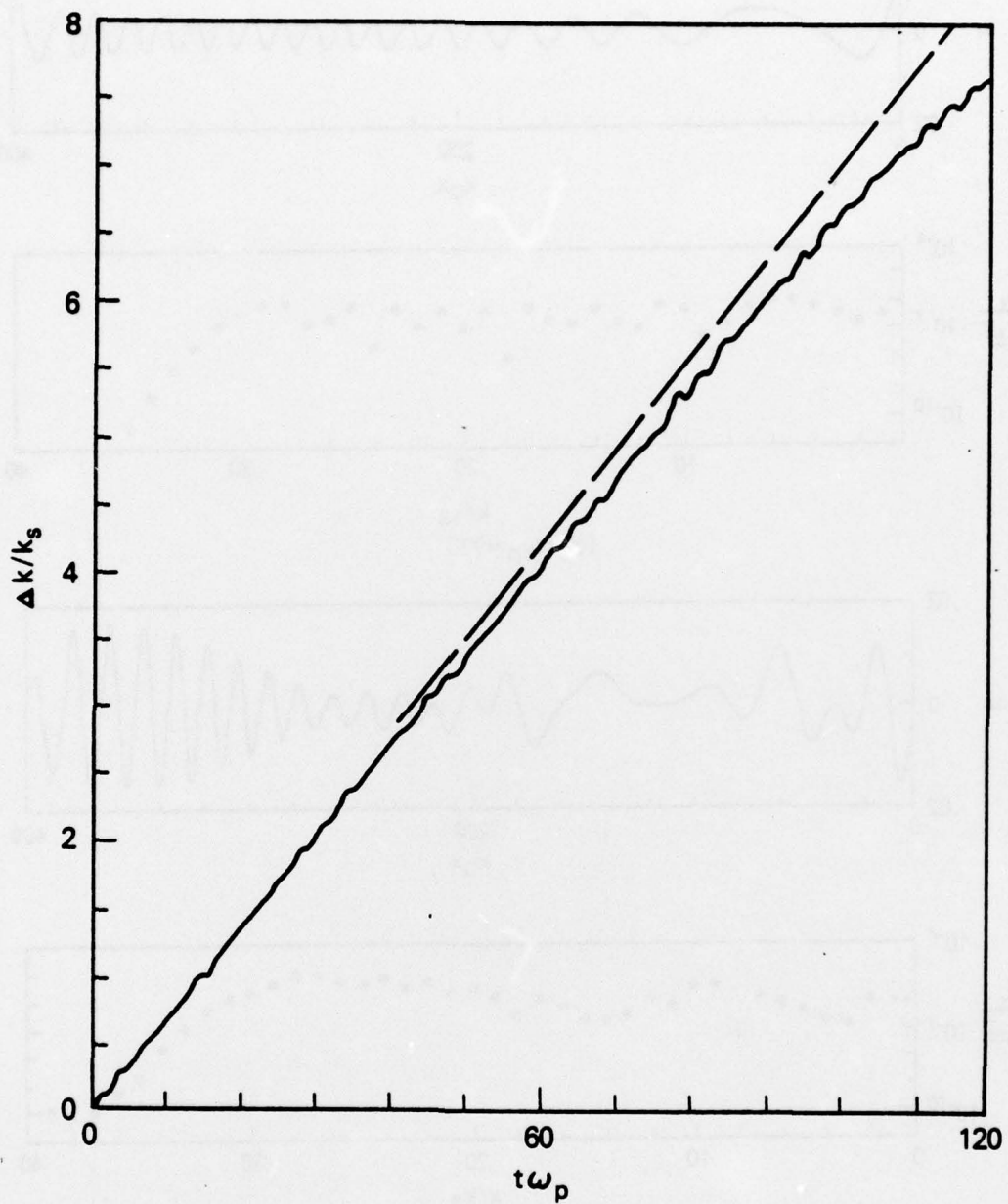
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Fig. 2 — The time history of the spread of Fourier components, $\Delta k = \langle k^2 - \langle k \rangle^2 \rangle^{1/2}$. The initial condition is that shown in Fig. 1(a).

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